Quantization of neutron in Earth's gravity

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Gravity is the weakest of all four known forces in the universe. Quantum states of an elementary particle due to such a weak field is certainly very shallow and would therefore be an experimental challenge to detect. Recently an experimental attempt was made by V. V. Nesvizhevsky et al., Nature **415**, 297 (2002), to measure the quantum states of a neutron, which shows that ground state and few excited states are $\sim 10^{-12}$ eV. We show that the energy of the ground state of a neutron confined above Earth's surface should be $\sim 10^{-37}$eV. The experimentally observed energy levels are 10^{25} times deeper than the actual energy levels it should be and thus certainly not due to gravitational effect of Earth. Therefore the correct interpretation for the painstaking experimental results of Ref. [1] is due to the confinement potential of a one dimensional box of length $L \sim 50\mu$m, generated from the experimental setup as commented before [11]. Our results thus creates a new challenge to the experimentalist to resolve the shallow energy levels of the neutron in Earth's gravitational field in future.

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The investigation of quantum phenomenon in gravitational field is certainly interesting and challenging [1, 2, 3, 4] due its weakness of strength. To get an idea of the weakness of gravitational force over other forces a quantitative estimation may be helpful. The the gravitational attraction of two neutrons separated by a distance r is $\sim 10^{-36}$ times weaker [5] than the Coulomb repulsion between the two electrons separated by the same distance. One therefore needs to be very careful while investigating the quantum effects of gravity. Neutron is a possible candidate on which quantum effects of gravity can be investigated because charge neutrality will eliminate electromagnetic force from our considerations.

The nature of the gravitational force F of Earth (except the strength) experienced by a neutron is same (long range and proportional to the inverse of the distance between the two agents) as that of the Coulomb force experienced by an electron in a Hydrogen atom. It is therefore expected that the nature of energy states of a neutron in the Earth's gravitational force will be similar to that of a Hydrogen atom with an infinite hard sphere core [6, 7]. We need to keep in mind that the neutron is above the Earth's surface, so we assume that the wave-function within the Earth is zero, i.e., $\psi(r) = 0$ for $r \leq R_\oplus$, where $R_\oplus$ is the Earth's radius (it is assumed that Earth is completely spherical). Since the neutron of mass m can not penetrate within the Earth, it will put an upper bound to the absolute value of the energy E_n of the discrete quantum states, which is $|E_n| \leq (\hbar^2/2m)R_\oplus^{-2} \approx 5.08 \times 10^{-37}$eV [6, 7]. Note that the states are $\sim 10^{-25}$ times less deeper than that obtained in the recent experiment by V. V. Nesvizhevsky et al., Nature **415**, 297 (2002).

Then the question arises that what is the reason of getting quantum states $\sim 10^{-12}$eV in the experiment? The correct interpretation for observing $\sim$peV (1peV= 10^{-12}eV) states in the experiment is the following. The experimental set up consists of a bottom mirror and a top absorber with a gap of approximate 50μm in between them. This can be considered as a problem of a particle in a one dimensional box [8] of length $L = 50\mu$m. In fact it has been commented before in Ref. [11], see the corresponding reply [12] also. For the present purpose we may neglect the dynamics of the neutron in the transverse direction. The energy levels for the neutron in the potential created by the box is $E_n = -(\hbar^2\pi^2 n^2/2m)L^{-2}$. The first few states for $L = 50\mu$m are respectively given by $E_1 \approx 0.082$peV, $E_2 \approx 0.3272$peV, $E_3 \approx 0.7362$peV, $E_4 \approx 1.309$peV, $E_5 \approx 2.0451$peV, $E_6 \approx 2.945$peV, $E_7 \approx 4.01$peV. Note that the experimentally obtained first four energy levels in Ref. [1] are comparable with the above obtained theoretical levels E_4, E_5, E_6 and E_7 respectively.

We then need to answer what is wrong with the previous theoretical prediction of Ref. [9], which shows that the discrete quantum levels due to Earth's gravitational force is $\sim$ peV? In fact it agrees with the experimental results [1]. The answer could be found partly in the potential $U(z) = mgz$ considered for the neutron above the Earth's surface. The other drawback is that the spherical symmetry of the problem due to central force has been completely ignored and thus the dynamics of the neutron in the z direction has be decoupled by assuming that in the transverse directions the particle is free. It is of course true that the potential $U(z)$ is approximately valid for $z \ll R_\oplus$. But for the neutron above the Earth the wave-function in principle would extend from earth's surface to infinity. Thus $U(z)$ is useless in this situation and instead should be replaced by spherically symmetric Newton's potential $V(r) = -GM_\oplus m/r$ [10], where G is the universal gravitational constant, $M_\oplus$ is the mass of the Earth and r is the distance of the neutron from the Earth's center. The potential within the Earth is as usual infinity because of the assumption that the probability of finding

the neutron within the Earth is zero. The ground state of the neutron on the Earth's surface in gravitational field is thus $E_{\rm g.s} \approx -(\hbar^2/2m)R_\oplus^{-2} \approx -5.08 \times 10^{-37}$eV [6, 7], since it is the maximum deep state of the neutron. The analytical calculation for all the excited states of the neutron will be in line with Ref. [7]. However analytical solutions for excited states are not important for our present purpose because they all will be even less than the ground state energy. The point that the deepest bound state (which is ground state) is $\sim 10^{-37}$eV is the most important message here. However one needs to think about the validity of the basic assumption in reality that the neutron wave-function within the Earth is zero. Because, the penetration of the quantum particle probability within the Earth will change the bound on the quantum energy levels of the neutron. But this is an issue which can be best resolved by experimental observation. We however have considered this assumption based on the experiment [1] The next immediate challenge to the experimentalist is to detect the quantum states due to Earth's gravity.

Our theoretical observation does not rule out the experimental detection of $\sim 10^{-12}$eV quantum states of a neutron but rather it gives a correct interpretation for the existence of peV states. The one dimensional box potential generated from the lower mirror and top absorber of length $\sim 50\mu$m dominates the gravitational potential on the Earth's surface. Gravitational force on the Earth is so weak that the quantum states of a neutron due to such force is $\sim 10^{-37}$eV, based on the assumption that neutron wave-function inside the Earth is zero. The experimental resolution power should be much higher than the present we have [1] in order to detect such quantum states.

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